

## Principles of the PhD Qualifying Exam

- The PHD Qualifying Exam in Department of Statistics consists of THREE parts, namely Part I, Part II, and Part III. Part I and II are written exam while Part III is an oral exam. Only the students who pass Parts I and II are allowed to take Part III, the Oral Examination. The candidate is considered successful when he/she passes all three parts.
- Ph.D. Qualifying exam is given twice a year each November and May. Students have to apply with their letter of application for the Qualifying Exam to the department by the end of October and April, respectively.
- The candidate failing to pass the Ph.D. Qualifying exam is given a second chance in the subsequent offering of the exam. Failure in the second attempt leads to the dismissal of the student from the Ph.D. program.

### Part I: Probability Theory & Theory of Statistics

**Aim/Objective of Probability Theory Exam:** The intend of this written examination is to have students intensify their ability to follow the ideas given in Probability Theory.

**Guidelines of Probability Theory Exam:** You may be asked to prove the specific results listed below as ‘Theorem,’ ‘Lemma’ or ‘Law’ or you may be asked to state results and summarize the proofs or you may be asked to solve or prove smaller exercises, which you may not have seen before.

#### Emphasized Topics:

- Probability measure,  $\sigma$ - algebras, measurability, random variables, distribution, expected value, Modes of convergence (almost sure (almost everywhere), in probability (stochastic convergence), in  $L^p$ , in distribution (weak convergence) and the relationship between them)
- Law of Large Numbers
- $\lambda$ - $\pi$  systems, Independence and dependence of random variables and of  $\sigma$  algebras, Monotone Class Theorem and related topics
- Kolmogorov zero-one Law
- Characteristic Functions and the inversion formula, Infinitely divisible distributions, compound Poisson, general form of infinitely divisible
- Tightness and the method for showing convergence in distribution, Continuity Theorem
- Central Limit Theorem
- Conditional Expectations: Definition and Properties
- Martingales, filtrations, stopping times, Martingale Convergence Theorem, The Up-crossing Lemma
- Uniform integrability, theorems combining uniform integrability and conditional expectation, theorems combining martingales, uniform integrability and almost sure convergence
- Probability Concepts in Stat 501 and Stat 502

**Related Courses:** Stat 601, Stat 602, Probability Concepts in Stat 501 and Stat 502.

**Suggested References:**

- Richard Durrett (2017) Probability: Theory and Examples, Duxbury Press.
- Patrick Billingsley (2012) Probability and Measure, Wiley series in probability and mathematical statistics.
- Erhan Çinlar (2011) Probability and Stochastics, Springer
- Hayri Körezlioğlu, Azize Bastıyalı Hayfavi and Yeliz Yolcu Okur (2018) Elements of Probability Theory, METU Press.
- George Casella and Roger L. Berger, Statistical Inference, Duxbury

**Guidelines of Theory of Statistics Exam:**

**Aim/Objective of Theory of Statistics Exam:** The aim is to evaluate student's theoretical knowledge needed to develop a methodology for estimation and assessing the statistical properties of estimators. This part includes advanced theorems and methodologies related with the following topics:

- Techniques of estimation: moments method; likelihood based estimation
- Likelihood construction
- Conditional likelihood
- Information matrix calculations
- Statistical properties of estimators
- Hypothesis testing: Score tests, Wald tests, likelihood ratio tests
- Construction of confidence intervals
- Statistical properties of hypothesis tests: MP tests, UMP tests

**Related courses:** STAT 501, STAT 502, STAT 603, STAT 604

**Suggested references:**

- Boos, D.D. and Stefanski, L.A. (2013) Essential Statistical Inference: Theory and Methods
- Wasserman, L. (2003) All of Statistics: A Concise Course in Statistical Inference
- Bain, L.J. and Engeldhart, M. (1994), Introduction to Probability and Mathematical Statistics"
- Casella, G. and Berger, R.L. Statistical Inference, Duxbury
- Mood, A.M., Graybill, F.A., Boes, D.C. (1974), "Introduction to the theory of statistics", McGraw-Hill Publishing USA
- Sahoo, P. (2013), Probability and Mathematical Statistics
- Hogg, R.V., McKean, J., and Craig, A.T, Introduction to Mathematical Statistics

## **Part II: Computational Statistics and Data Analysis**

**Aim/Objective of Probability Theory Examination:** The aim of this part is to assess student's computational knowledge and ability for conducting simulation studies and data analysis. Topics include

- Random number generation
- Monte Carlo methods for statistical inference
- Resampling methods, Bootstrap, Jackknife
- Writing your own functions,
- Using statistical software for statistical inference
- Data Analysis

**Related Courses:** STAT 554

### **Suggested references:**

1. Gentle, J.E. (2009) Computational Statistics
2. Givens, G.H. and Hoeting, J.A. (2012) Computational Statistics

### **References for R:**

- Adler, J. (2010) R in a nutshell. O'Reilly Media, Inc. California. [E-copy available at METU]
- Gardener, M. (2012) Beginning R. J. Wiley & Sons, Indianapolis, Ind. [E-copy available at METU]
- İlk, Ö. (2015) R Yazılımına Giriş, 2. baskı, ODTÜ Yayıncılık.
- Matloff, N. (2011) The art of R programming. No Starch Press, San Francisco. [E-copy available at METU]
- Muenchen, R.A. (2009) R for SAS and SPSS Users. Springer Science & Business Media, NY. [E-copy available at METU]
- Teetor, P. (2011) R Cookbook. O'Reilly Media, Inc. California. [E-copy available at METU]
- Teetor, P. (2011) 25 recipes for getting started with R. O'Reilly Media, Inc. California. [E-copy available at METU]

### **References for MATLAB:**

- Attaway, S. (2009) MATLAB: a practical introduction to programming and problem solving. Butterworth-Heinemann, Boston. [E-copy available at METU]
- Biran, A. and Breiner, M. (1999) MATLAB 5 for Engineers, Addison-Wesley.
- Martinez, W.L. and Martinez, A. R., (2002) Computational Statistics Handbook with MATLAB, Chapman & Hall/CRC, Boca Raton.
- Part-Enander, E. and Sjöberg, A. (1999) The MATLAB 5 Handbook, Addison-Wesley.
- Spencer, R. and Ware, M. (2006) Introduction to MATLAB. Brigham Young University, Provo.

### **Part III: Oral Examination**

**Aim/Objective of the Oral Examination:** The students will be examined on the Fundamental ideas of Theory and Application of Statistics and they will be encouraged to talk about their future research plan's and the obstacles they may encounter during the process. The student's ability and potential to conduct a doctorate level research will be measured.

#### **Grading Policies of the Examination:**

1. Probability Theory & Theory of Statistics The candidates will be asked to answer two questions out of three in each exam. The exam will be graded out of 100 and 60 percent of the result of this Part I will be calculated.
2. Computational Statistics & Statistical Computing. The candidates will be asked to answer two questions out of three. The exam will be graded out of 100 and 40 percent of the result of this Part II will be calculated.

The candidates are considered successful if their Total grade of the Written Exam (Part I + Part II) is 70.

In addition, if the student is successful in any of the Parts (i.e. if s/he gets 70 out of 100 in Part I or Part II) of the written exam, s/he will be exempt from the Part on which s/he has passed. In the subsequent exam, s/he will be expected to pass the part (which is 70 out of 100) she has failed with and continue the process accordingly.